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On-Line Version of the Eigensystem Realization Algorithm Using Data Correlations

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An on-line (recursive) version of the eigensystem realization algorithm using data correlations is developed based on the QR decomposition. The method can be used to estimate a theoretically unbiased minimum-order realization from decay response data. The proposed method is an improvement on other on-line time domain identification techniques in that the minimum-order realization permits overspecification of the mathematical model, removing simultaneously the problems of a priori model order selection and possible computational difficulties with identified spurious modes. Time varying modal parameters can also be tracked through the use of a forgetting factor to weight the most recent points. The method is demonstrated on a number of simulated data sets corrupted with measurement noise.

Introduction

THE majority of modal tests are carried out with the assumption that the parameters of the system under test remain constant. Therefore, it is usual to acquire all of the test data and then to estimate the modal parameters using all of the data together. Such a procedure is known as off line. There are many methods available to perform such an analysis based either in the time domain or the frequency domain.[1] The modal analysis community makes frequent use of identification methods that curve-fit decay responses in the time domain.

Most time domain system identification methods used to identify modal parameters from impulse responses have the least squares (LS) minimization of a difference equation [autoregressive (AR) model] technique as the fundamental element.[2,3] Such methods have to be employed with a large amount of model order overspecification (i.e., fit a model that is larger than the number of modes that are actually present) to obtain unbiased modal parameter estimates. This process results in the user having to distinguish between the system and spurious modes. The eigensystem realization algorithm (ERA)[4] is an improvement compared to the majority of other methods in that a minimum realization is performed, theoretically enabling all of the spurious modes to be eliminated during the estimation process. However, the LS technique is still the fundamental element,[3] resulting in the need for large matrices (implying a large amount of overspecification) to eliminate all of the bias.

The ERA using data correlations (ERA/DC)[5] ignores the data correlations likely to cause biased estimates and, therefore, requires less model order overspecification. The computational requirement is consequently smaller compared to the ERA. Previous investigations[6,7] have shown ERA/DC to perform well in comparison to other identification methods.

There are a number of structural systems where the assumption of stationary modal parameters is not valid including: aeroelastic systems where changes in speed or altitude change the frequencies and dampings, and space rockets where the mass of the system changes rapidly. There is also a requirement in parts of the aerospace industry worldwide to perform large deflection step release tests on aircraft wings; the damping in such tests has been found to vary as the decay response progresses. Finally, the impulse testing of space structures results in extremely lightly damped responses during which the modal parameters may change.

There are two approaches that can be employed to track the changing parameters of such systems. One approach is to divide the data up into small segments and to curve fit each segment as if it contains stationary modal parameters. The problem with this method is that the segments may have to be very small to track rapidly varying systems, which tends to make the estimates inaccurate. Also, no account is made of the estimates found from previous data segments.

The alternative approach is to use so called on-line (or recursive) techniques. On-line system identification methods are formulated in the time domain, the data are considered at each time instance sequentially, and estimates are updated at each time instance. Although there has been a lot of work devoted to developing such techniques in the signal processing and control fields,[8,9] there has been little application of such methods to time varying structural systems.[10–13] Most work has concentrated on the analysis of input–output data sets. To this author's knowledge, the work by Davies[14] was the first to show how the classical recursive LS algorithm can be applied to decay response data through the use of an AR model, and this resulted in an on-line version of the LS complex exponential algorithm. It is straightforward to use the same approach for all of the other time domain methods based on a LS minimization.[15] Note that although the procedure to obtain the difference equation parameters is on line, the estimation of the frequencies and dampings at each time step requires an eigensolution, which has to be calculated explicitly.

The development of on-line versions of those methods employing a minimum order realization has not proved so straightforward, as

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the singular value decomposition (SVD) is used in both the ERA and ERA/DC. At present there is no exact way of finding the exact SVD, or eigenvalue solution, of an updated matrix, although a number of approximate methods exist.[16] A number of on-line versions of the ERA have been developed[17,18] based on the Gram–Schmidt procedure for updating the QR decomposition; however, minimum-order realizations are not obtained. The use of an overspecified model for all of the mentioned on-line methods often leads to stability problems with the spurious modes[19] and is not recommended.

An off-line QR version of the ERA/DC is formulated, enabling an on-line implementation of the method to be developed. The technique produces a minimum-order realization with theoretically unbiased modal parameter estimates. Although an eigenvalue solution is still required at each time step, the minimum-order solution means that no spurious modes need to be estimated, thereby eliminating possible convergence problems. A forgetting factor is also introduced to the formulation, enabling time varying parameters to be tracked. The method is illustrated with a number of simulated examples.

QR Version of ERA/DC

Both the ERA and ERA/DC were developed originally using the SVD. However, there is no reason why other decompositions such as the QR decomposition[20,21] cannot be used. If the ith impulse response at time $(k+1)$ due to the jth input is denoted as y_{ij}, then all of the responses may be written as

$$Y_{k+1} = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,NI} \\ y_{2,1} & y_{2,2} & \cdots & y_{2,NI} \\ \cdot & \cdot & \cdots & \cdot \\ y_{NO,1} & y_{NO,2} & \cdots & y_{NO,NI} \end{bmatrix}_{k+1} = CA^k B \tag{1}$$

where Y is an $(NO \times NI)$ data matrix, A is the $(M \times M)$ system matrix, and C and B are the $(NO \times M)$ measurement and $(M \times NI)$ input matrices, respectively. M is the system order (twice the number of modes), and NO and NI are the number of measurement stations and input cases, respectively. The identification method is used to find a realization of the A, B, and C matrices. The eigenvalues of system matrix A lead to the frequencies and dampings, and matrices C and B can be used to estimate the mode shapes and modal participation factors.

Defining the block Hankel matrix H_j as

$$H_j = \begin{bmatrix} Y_{j+1} & Y_{j+2} & \cdots & Y_{j+\eta+1} \\ Y_{j+2} & Y_{j+3} & \cdots & Y_{j+\eta+2} \\ \cdot & \cdot & \cdots & \cdot \\ Y_{j+\xi+1} & Y_{j+\xi+2} & \cdots & Y_{j+\xi+\eta+1} \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \cdot \\ CA^{\xi} \end{bmatrix} A^k [B \quad AB \quad \cdots \quad A^{\eta} B] = VA^k W \tag{2}$$

then correlation terms S_j may be written as

$$S_k = H_k H_0^T = VA^k W W^T V^T = VA^k W_c$$

Now, the correlation matrix U_{q+k} is defined such that

$$U_{q+k} = \begin{bmatrix} S_{q+k} & S_{q+k+r} & \cdots & S_{q+k+\beta r} \\ S_{q+k+r} & S_{q+k+2r} & \cdots & S_{q+k+(\beta+1)r} \\ \cdot & \cdot & \cdots & \cdot \\ S_{q+k+\alpha r} & S_{q+k+(\alpha+1)r} & \cdots & S_{q+k+(\alpha+\beta)r} \end{bmatrix} = \begin{bmatrix} V \\ VA^r \\ \cdot \\ VA^{\alpha r} \end{bmatrix} A^{q+k} [W \quad A^r W \quad \cdots \quad A^{\beta r} W] = V_\alpha A^k A^q W_\beta = V_\alpha A^k W_q \tag{3}$$

where the q term is set so that those correlations likely to cause bias are not included in the curve fit and the r term ensures that the correlation blocks do not overlap.[5,7]

Setting U_q as an $(\gamma \times \delta)$ matrix with $\gamma \geq \delta$, then making use of the QR decomposition[22] gives

$$U_{q(\gamma\times\delta)} = Q_{(\gamma\times\gamma)} R_{(\gamma\times\delta)} \tag{4}$$

where the upper a rows of R are upper triangular and Q is orthogonal. If U_q has rank $m < \delta$, then Eq. (4) takes the form

$$U_q = [Q_1 \vdots Q_2] \begin{bmatrix} R_\gamma \vdots R_\delta \\ \cdots\cdots\cdots \\ 0 \vdots 0 \end{bmatrix} = Q_1 [R_\gamma \vdots R_\delta] = Q_{1(\gamma\times m)} R_{1(m\times\delta)} \tag{5}$$

where R_γ is upper triangular. The pseudoinverse of matrix R_1 can be defined as

$$R^{\#} = R_1^T \left(R_1 R_1^T\right)^{-1} \tag{6}$$

and, thus, the pseudoinverse of U_q is found as

$$U^{\#} = R^{\#} Q_1^T \tag{7}$$

hence,

$$U_q U^{\#} U_q = U_q \tag{8}$$

It can also be shown that

$$S_{q+k} = E_\alpha^T U_{q+k} E = E^T V_\alpha A^k W_q E_\beta \tag{9}$$

where the E matrices contain identity and null matrices in the form $[I \; 0 \; \cdots \; 0]$. Now, expanding Eq. (9) with Eqs. (2–8) leads to the realization

$$S_{q+k} = E_\alpha^T \tilde{Q} \left(Q_1^T U_{q+1} R^{\#}\right)^k \tilde{R} E_\beta \tag{10}$$

where $\tilde{Q} Q_1^T = I_{(\gamma\times\gamma)}$ and $R^{\#} R = \tilde{I}_{(\delta\times\delta)}$. Although $\tilde{Q}$ is not unique, this is not a problem as there are an infinite number of realizations. Comparison of Eqs. (9) and (10) enables the matrices A, V_α, and W_q to be found such that

$$A = Q_1^T U_{q+1} R^{\#} \tag{11}$$

$$V_\alpha = E_\alpha^T \tilde{Q} \tag{12}$$

and

$$W_q = \tilde{R} E_\beta \tag{13}$$

The eigensolution of matrix A gives the frequencies and dampings, and use of Eqs. (1) and (2) leads to the mode shapes and modal participation factors.

The Q and R matrices can be truncated to ignore the diagonal terms of R that are considered to be zero (in exactly the same manner that the SVD is used in the off-line ERA and ERA/DC methods); hence, Eq. (10) is a minimum-order realization [i.e., matrix A has reduced dimensions $(m \times m)$]. The ability to produce a minimum-order realization based on the QR decomposition is the key to the on-line ERA/DC formulation developed in the next section. It does not matter how many modes are taken in the initial model (defined by the dimensions of the U matrices) as the preceding realization enables, theoretically, all spurious modes to be eliminated through the truncation of the Q and R matrices.

On-Line Implementation of ERA/DC

As the preceding realization is formulated in terms of the QR decomposition, we can take advantage of the exact updating schemes that exist.[22] Considering the single input/single output case for simplicity, then the $(\gamma \times \delta)$ U_q matrix defined with $NI = NO = 1$ and $\xi = 0$ in Eqs. (1–3) can be updated to include the $(N+1)$th data point using the expression

$$(U_q)_{N+1} = (U_q)_N + \Phi_{N+1} \Psi_{N+1}^T \tag{14}$$

where

$$\Phi_{N+1} = \mathrm{diag}\,[y_{N+1}, y_{N+1}, \ldots, y_{N+1}] \tag{15}$$

and

$$\Psi_{N+1}^{T} = \begin{bmatrix} y_{q+N+1} & y_{q+N+2} & \cdots & y_{q+N+\beta+1} \\ y_{q+N+2} & y_{q+N+3} & \cdots & y_{q+N+\beta+2} \\ \cdot & \cdot & \cdots & \cdot \\ y_{q+N+\alpha+1} & y_{q+N+\alpha+2} & \cdots & y_{q+N+\alpha+\beta+1} \end{bmatrix} \tag{16}$$

Note that the dimensions of the U matrix and, hence, the Q and R matrices remain constant, unlike the previous implementations of the on-line ERA where the matrix dimensions changed at each time instant. Therefore, one advantage of the new approach is that the initial overspecified model order remains the same and the realization is updated at each time instant.

Given the QR decomposition for N time points, $(U_q)_N = Q_N R_N$, the updated decomposition $(U_q)_{N+1} = Q_{N+1} R_{N+1}$ is computed[22] making use of a number of row and column Givens rotations, denoted by J and G, respectively, as follows.

1) Find the matrix W such that

$$W = Q_N^T \Phi_{N+1} \tag{17}$$

2) Find Givens rotations such that

$$J_1^T \quad \cdots \quad J_{M-1}^T W \quad \text{is upper triangular} \tag{18}$$

3) Find

$$P = J_1^T \quad \cdots \quad J_{M-1}^T \left(R_N + \Phi_{N+1} \Psi_{N+1}^T\right) \tag{19}$$

where P is upper Hessenburg.

4) Find

$$R_{N+1} = G_{M-1}^T \quad \cdots \quad G_1^T P \tag{20}$$

such that R_{N+1} is upper triangular.

5) Find

$$Q_{N+1} = Q_N J_{M-1} \quad \cdots \quad J_1 G_1 \quad \cdots \quad G_{M-1} \tag{21}$$

Having updated the QR decomposition, the latest estimates of the realization can be made using the updated U_{q+1} matrix in Eqs. (9–11). The initial estimates are found by taking the minimum number of data points required to produce an estimate. Note that, as with all other on-line identification methods, the eigensolution of system matrix A has to be found to estimate frequencies, dampings, and mode shapes at each times step.

An alternative formulation is to set up the correlation matrix with $\alpha = \beta = 0$. A careful choice of the block Hankel matrix, along with that of the delay term q, will result in a correlation matrix that does not contain any bias causing terms. The motivation for this second formulation is that the equivalent version of the updating process in Eq. (14) results in a much simpler rank one addition, requiring less computational effort. One disadvantage, however, is that the correlation matrix becomes square, thereby reducing the flexibility to dictate how many different correlation terms are included in the fit. Although the on-line version of ERA/DC requires more computation than the on-line version of ERA to identify parameters from the same size of matrices, it is envisaged that in practice much smaller matrices will be required for the ERA/DC approach, as has been found for the off-line formulations.[6] Further work is required to establish the computational benefits of the on-line ERA/DC method.

With the ability to find a minimum-order realization, the elimination of the spurious modes avoids possible convergence problems that are sometimes experienced in on-line identification when an overspecified model is used. Any initial model order overspecification is beneficial in that it aids an unbiased solution. The determination of the number of modes in the system can be based on the QR decomposition rather than having to guess before beginning the identification process.

Implementation of Forgetting Factors

One advantage of using an on-line rather than an off-line formulation is that it is possible to track changes in the system parameters through the inclusion of a forgetting factor to give a weighting to the most recent data points. The smaller that the forgetting factor, denoted here as λ, is, the greater the weight given to the most recent data. However, too small a value of λ will result in nonsmooth results due to the effect of noise on the data. Thus, its use is a compromise between being able to follow rapid changes in the system parameters and being able to smooth out the effect of noise.[8]

The simplest way to include such an approach in the on-line version of ERA/DC is to weight the data matrices by forgetting factor λ with $0 < \lambda \leq 1$ such that

$$(U_{q+1})_{k+1} = \lambda (U_{q+1})_k + \Phi_{k+1} \Psi_{k+1}^T \tag{22}$$

and

$$(U_q)_k = \lambda Q_k R_k = Q_k \lambda R_k \tag{23}$$

It has been shown that the best way to implement such an approach is to use an adaptive forgetting factor. When the system parameters are almost stationary, λ is set close to unity, and when the parameters are changing rapidly, the value of λ is reduced so that only the most recent data points have greatest influence on the estimates. A number of different methods have been proposed to automate such a scheme.[23–25]

In this work, a simpler approach is used,[26] based on moving window values of the parameter values to determine suitable values of the forgetting factor. If the parameter values throughout the window are the same, the slope of a line drawn through the slope is zero, and λ should be set to one. If the system is changing rapidly, the slope will increase and λ should be reduced. A rule that meets these criteria is

$$\lambda = e^{-\Omega |S|} \tag{24}$$

where S is the slope of the sliding mean parameter values. Ω has to be predefined and set so that λ lies mainly in the region $0.95 < \lambda < 1.0$. Some experimentation with the value of Ω can be required to achieve a suitable behavior. If required, the standard deviation of the parameter estimates can be monitored and λ increased if the standard deviation becomes greater than some predefined level.

Application of the Method

As an example of the use of the on-line ERA/DC, consider a two-degree-of-freedom system with natural frequencies 10 and 15 Hz with damping ratios 1 and 2%, respectively. The data were corrupted with random noise with a 10% noise/signal ratio, where the noise/signal ratio is defined as the rms value of the noise divided by the rms value of the signal. The data were sampled at 50 Hz. As the frequency values are generally estimated very well, the results shown will concentrate on the estimated damping ratios as these are the most sensitive to errors in the estimation process. This section is not intended to be an exhaustive statistical investigation, merely an illustration of how the method can be used.

Figure 1 shows the estimates using the smallest possible U_q matrix (4×4) with a zero q value, which is essentially the form used by all of the methods not employing a minimum-order realization. The

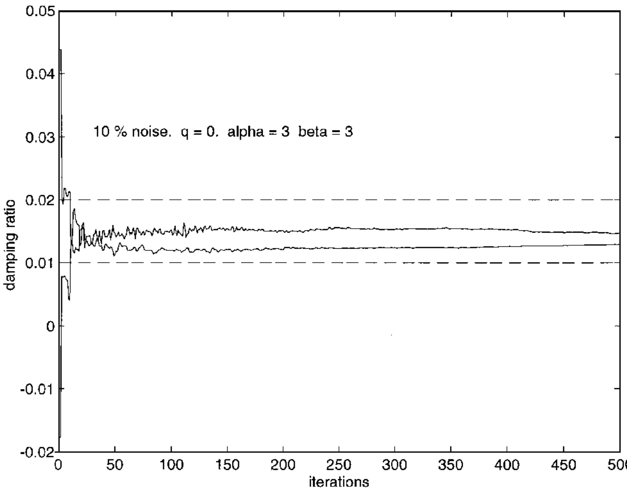

Fig. 1 Damping ratio estimates, 10% noise, $q = 0$, $\alpha = 3$, $\beta = 3$.

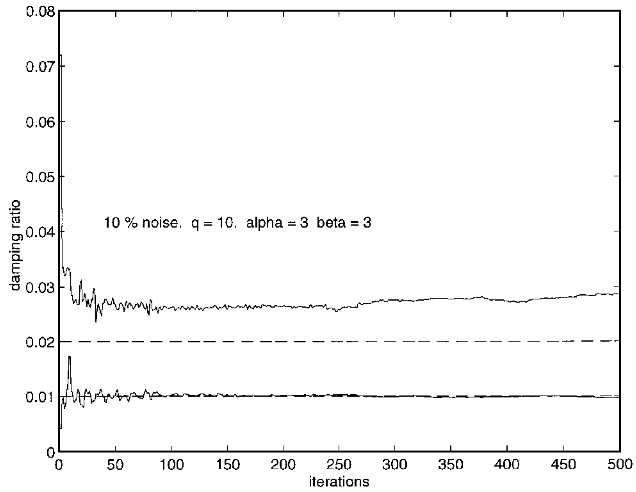

Fig. 2 Damping ratio estimates, 10% noise, q = 10, α = 3, β = 3.

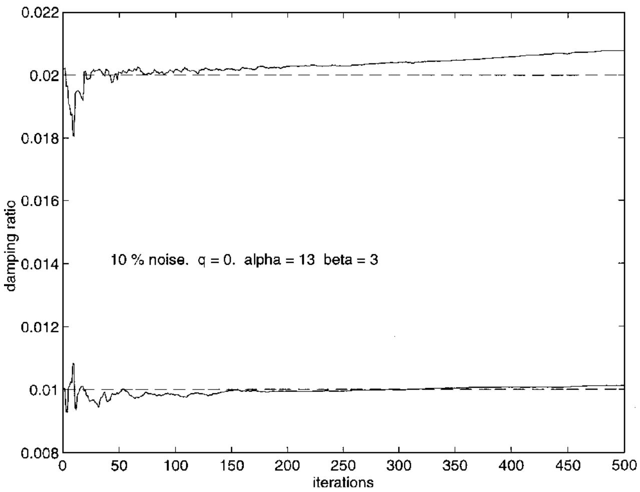

Fig. 3 Damping ratio estimates, 10% noise, q = 0, α = 13, β = 3.

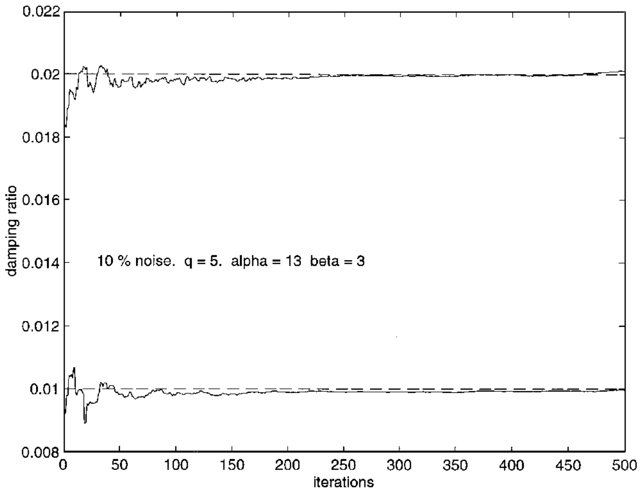

Fig. 4 Damping ratio estimates, 10% noise, q = 5, α = 13, β = 3.

damping ratios contain a large amount of error. If the correlation lag term is then increased so that $q = 10$, then it can be seen in Fig. 2 that the estimates are better although there are still problems with the 15-Hz, 2% mode.

One of the benefits of the ERA/DC is that the number of correlation terms used in the fit is not dependent on the order of the model considered. Thus, the increase in the number of rows of the U_q matrix in Fig. 3 gives good results, although the estimation of the 15-Hz mode becomes a problem when its response becomes dominated by the noise. This difficulty can be eliminated, as in Fig. 4, through the choice of a suitable value of q. Comparison of Figs. 2, 5, and 6

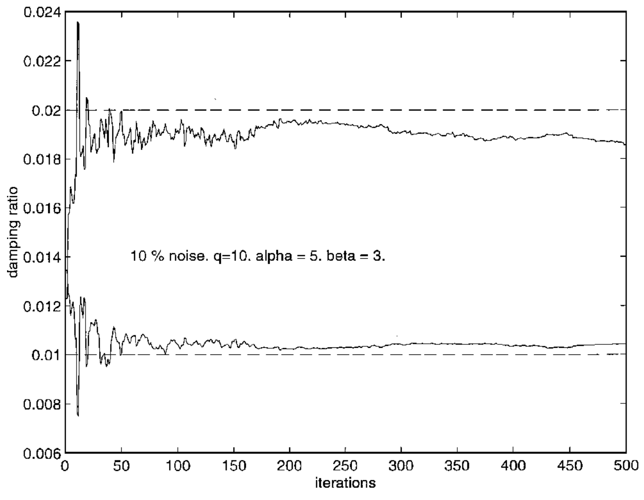

Fig. 5 Damping ratio estimates, 10% noise, q = 10, α = 5, β = 3.

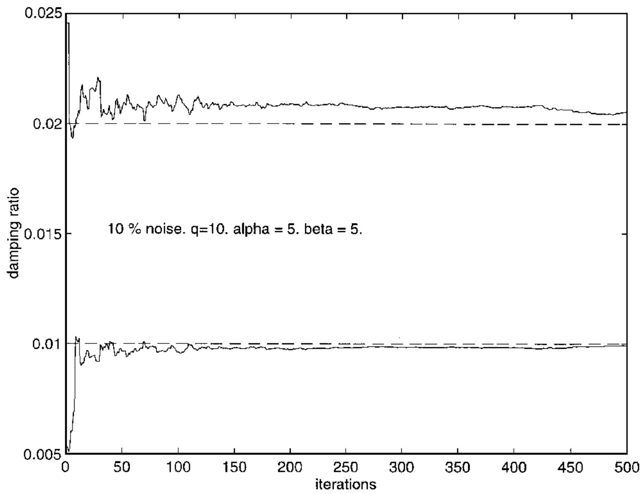

Fig. 6 Damping ratio estimates, 10% noise, q = 10, α = 5, β = 5.

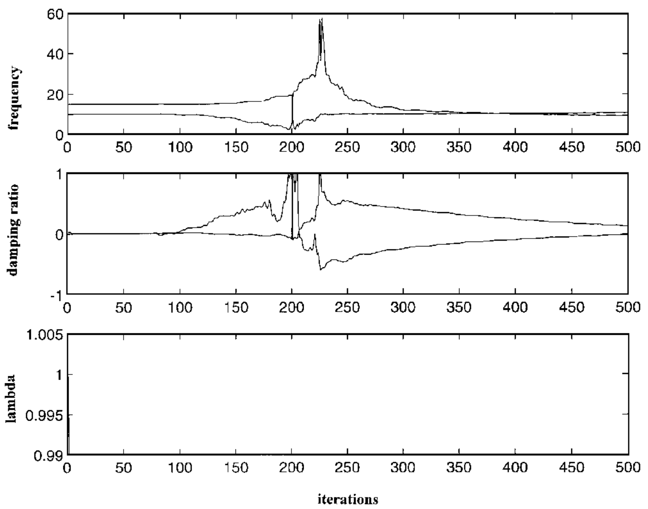

Fig. 7 Time varying case, constant forgetting factor; frequency and damping ratio estimates: 2% noise, q = 10, α = 7, β = 5, λ = 0.99.

shows how the decreased error due to an increase in the number of rows can be further enhanced by using an overspecified model with an increase in the value of β.

Finally, as an example of the use of the method for the time varying case, consider a system with initial frequencies 10 and 15 Hz with damping ratios 1 and 0.5%, respectively, which suddenly change after 100 time intervals to 8 and 12 Hz with the same damping ratios. Measurement noise of 2% was added to the signal. Figures 7, 8, and 9 show the estimated frequency and damping values for

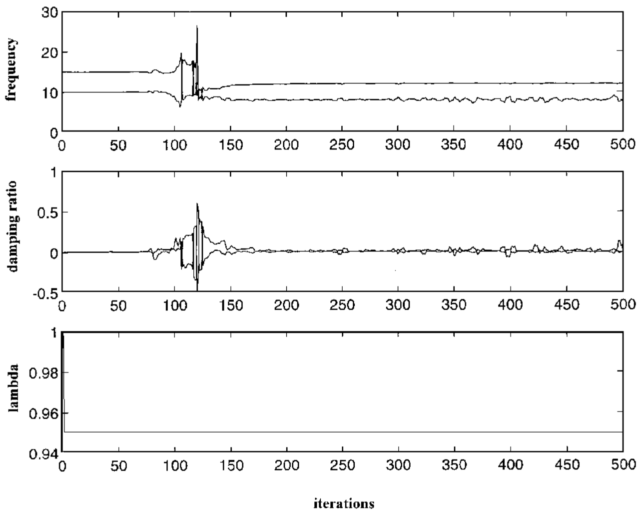

Fig. 8 Time varying case, constant forgetting factor; frequency and damping ratio estimates: 2% noise, $q = 10$, $\alpha = 7$, $\beta = 5$, $\lambda = 0.95$.

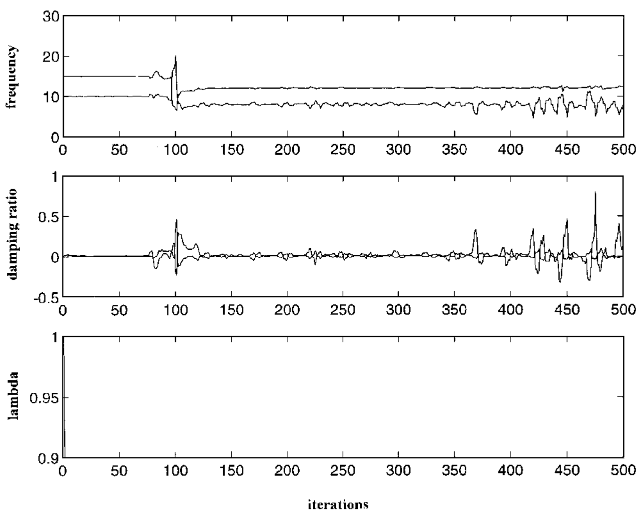

Fig. 9 Time varying case, constant forgetting factor; frequency and damping ratio estimates: 2% noise, $q = 10$, $\alpha = 7$, $\beta = 5$, $\lambda = 0.9$.

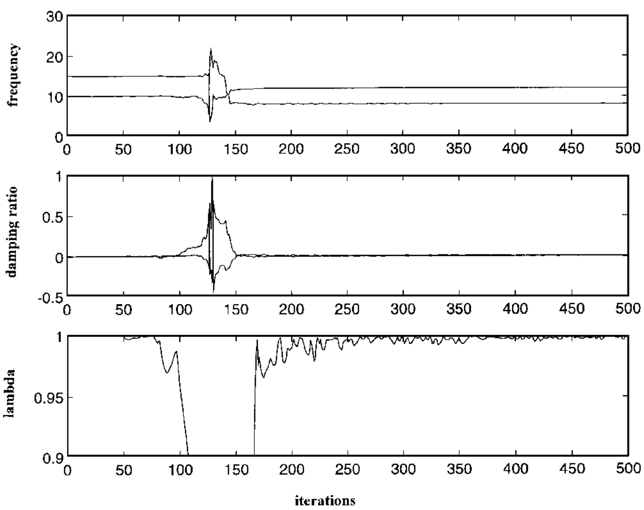

Fig. 10 Time varying case, variable forgetting factor; frequency and damping ratio estimates: 2% noise, $q = 10$, $\alpha = 7$, $\beta = 5$.

constant forgetting factors of 0.99, 0.95, and 0.9, respectively. Figure 10 shows the estimates for using the adaptive forgetting factor scheme described with a minimum value for the forgetting factor set at 0.9.

It can be seen that the use of a constant forgetting factor does not solve the problem of achieving a compromise between stabilizing in a short enough time while still smoothing out the effect of noise. However, when the adaptive approach is implemented, the abrupt change in the parameters results in a drop in the value of the forgetting factor, enabling the changes to be monitored quickly before the rising value of the forgetting factor smoothes out the estimates. Note that almost exact estimates were obtained once the estimates had stabilized. Further work is continuing to determine the best scheme for applying the adaptive forgetting factors.

Conclusions

An on-line version of the ERA/DC modal parameter identification method has been developed based on the QR decomposition. The method enables unbiased modal parameters to be found with a minimum-order realization, thereby allowing an overspecified mathematical model to be used without facing the convergence difficulties often found when performing on-line system identification with spurious modes included in the estimation process. Time varying systems can also be identified by using an adaptive forgetting factor to weight the most recent data points. The method has been demonstrated successfully on a number of simulated data sets.

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